FI=HLML04

MORE NOTATION ON SUBMODELS

ONE-WAY ANCOVA (ANALYSIS OF COVARIANCE) WITH RANDOM EFFECTS[1]

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Purpose of the Model

With this model you estimate effects of L-1 predictors while controlling for L-2 context. The L-2 context is assumed to have random effects. We will have no predictors at L-2. But we will control for L-2 differences on the mean outcome.

A typical ANCOVA Such a model can answer the following question:

> Does an individual-level predictor have a significant impact on pooled within-unit outcome scores, after I control for group context?

Here, you focus primarily on the impacts of the individual-level predictor.

For example, in a sample of police officers from different police departments you find effects of officer age on cynicism about police work, to take a Niederhoffer type example. Does such an impact hold up after I control for impacts of different department contexts. Context might "wash out" an individual level effect if average department ages vary significantly.

HLM Equations

From B&R p. 19:

Level 1: YIJ = B0J + B1J(XIJ - X..) + RIJ   Eq. 4.01

Level 2: B0J = G00 + U0J   Eq. 4.02

B1J = G10   Eq. 4.03

Combined model:

YIJ = G00 +G10(XIJ - X..) +U0J + RIJ   Eq. 4.04

[1] For a pretty decent introduction to ANCOVA, see Blalock (1979) Social statistics (Rev. 2nd ed.), Chapter 20.

The same model applies for more than 1 L-1 predictor; we just have additional versions of Eq. 4.03. For example:

B2J = G20
B3J = G30

and so on.

Notice the following features about these models.

Each L-1 predictor has been centered by the **grand** mean. We do this, I think, because we are not going to be allowing the slope for the predictor to vary, and because this is what happens in "classic" ANCOVA.

The true outcome mean for each group (B0J) has only a random effects predictor: the unique effects of each different L-2 unit (U0J). We include no fixed effect predictors.

We are NOT allowing the slope on the L-1 predictor (B1J) to vary at all. **If we are not allowing it to vary we are constraining the variance of B1J, i.e., the variance of T11, to zero**. Conceptually, we are saying that the effects of our L-1 predictor (e.g., age) are the same in every L-2 unit.
In OLS regression, this assumption is called "homogeneity of regression." It is something we can test empirically with HLM, and see whether or not it is actually warranted.

The variance of RIJ (sigma squared) is residual, pooled, within-unit variance, after controlling for the effects of X1.

The variance of U0J (T00) is **un**conditional.

Look at the attached printout. The outcome examined here is Q21, the respondent's perceived chances of being attacked in his/her neighborhood. Respondents are the 870 Minneapolis-St. Paul residents interviewed around 24 small commercial centers in 1982. Scores on the outcome range from 0 (no chance) to slight (1), fair (2), to good (3). The predictor used is age, logged, then z scored.

Interpret:
- G00 (careful here!!!)
- reliability
- each fixed effect and the associated hypothesis test
- Chi square test of T00
- variance of RIJ
- can you estimate the residual intraclass correlation?

ONE WAY ANCOVA WITH RANDOM EFFECTS AND LEVEL 2 COVARIATE(S)

A modest elaboration of the model allows the introduction of Level-2 covariates. So now the equations become:

Level 1: same as Eq. 4.01

Level 2: $B0J = G00 + GO1(W1) + U0J$ Eq. 4.05

$B1J = G10$ Eq. 4.06

Combined model:

$YIJ = G00 + G01(WJ) + G10(XIJ - X..) + U0J + RIJ$ Eq. 4.07

Substantively, you may have <u>either</u> a L-1 predictor as your central measure of interest, and a L-2 predictor as a covariate, or <u>vice versa</u>. For example, you might want to see the effects of age on perceived risk, controlling for levels of crime, using the crime stratification variable. Or you might be interested in the effects of neighborhood crime, as stratified, on perceived risk controlling for respondent age.

With this elaboration

Note how your interpretation of G00 changes yet again, because you have more predictors (Remember: an intercept always tells you about Y when all Xs, however many of them there are, are at 0.

U0J has been residualized with respect to W, your L-2 predictor. The change in T00 tells you how much W has reduced it.