FI=HLML06

HLM, Growth Curves, and Change

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Purpose

Particularly in psychology, researchers have had a lot of arguments since the early 1980s, about how to model "growth." For our purposes, we can simply translate "growth" into changes over time where you have more than two time frames of measurement.[1] For a good discussion see the references cited for Rogosa.

In criminal justice, concerns with change over time translate clearly into two areas of study:

- changes in crime rates or victimization rates over time
- modeling offenders' or delinquents' careers as they unfold over time

There may be other areas of interest as well.

HLM radically changes how we approach these issues, transforming how we conceptualize and analyze.

The Two Levels

Our two levels have changed.

Now, Level-1 refers to **the time of the observation** made.

Level-2 refers now to the **person** him/herself, or to the community or state or nation, in the case of crime rates or victimization rates.

> At Level-2, each person's development is represented by an individual growth trajectory that depends on a unique set of parameters. These individual level growth parameters become the outcome variables in a Level-2 model, where they may depend on some person level characteristics.[B&R, p. 131]

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[1] The reason you need more than two points in time is because with only two points in time you cannot distinguish between linear vs. curvilinear vs. other polynomial trends.

So what emerges from L-1 is a description of how things start out, and how they change, for each individual person or community. At L-2 you now want to predict those parameters that **describe** those changes.

HLM vs. MRM

On p. 133 B&R contrast HLM growth models with multivariate repeated measures. The most important point from our perspective is probably that you need not have evenly spaced measurement intervals, and need not have observations of every unit at every point in time for HLM to work reasonably well; this does not work in repeated measures.

Model Notation

At L-1

$$Y_{ti} = \pi_{0i} + \pi_{1i}a_{ti} + \pi_{2i}a^2_{ti} + \ldots \pi_{Pi}a^P_{ti} + e_{ti}$$

where:

Y_{ti} = score on Y of individual i at time t

π_{0i} = the unit's estimated 'true' ability or 'true' score on the Y attribute at time = 0 or age = 0 (a=0). Again: this is the intercept

π_{1i} = empirical Bayes best estimate of the 'true' linear growth rate or change rate over the t time periods, per unit time. It represents "expected [linear] change during a fixed unit of time" [B&R p. 134].

π_{1i} = empirical Bayes best estimate of the 'true' quadratic growth rate or change rate over the t unit time periods, per unit time. This is just like adding a quadratic predictor in OLS (X * X), except that here you are multiplying the "age" measure as your predictor

π_{Pi} = empirical Bayes best estimate of the Pth power growth rate per unit t time.

e_{ti} = portion of Yit that is not modeled; conditional error or residual variance. Assume these are independent across ts within individuals, and normally distributed. If there are a lot of ts, however, you may need to worry about serial autocorrelation. See comments top of

p. 132.

Note that as you add additional predictors (a) you are taking age and making it to a higher and higher power until you get up to a^{P}_{ti}

In other words the different predictors are really the same predictor, raised to different powers.

At L-2, the person level model, we predict the outcomes from L-1.

Here your predictors are X1, X2 .... Xq

Modeling the intercept:

π_{0i} = B00 + B01(X01i) + ... B0q(X0qi) + r0i

and modeling the slope parameters such as π_{1i}

π_{i} = B10 + B11(X11i) + ... B1q(X1qi) + r1j

Note how this looks like our usual L-2 model, except we have substituted Bs for Gs, and rs for Us.

The following translation table may help

Standard Two Level Model	Growth Model
L-1	
Intercept: B0J	π_{0i}
Slope: B1J	π_{1i}
Predictor: X1, X2...	a^1, a^2, a^3,... a^P
Residual: RIJ	e_{ti}
L-2	
Intercept: G00	B0J
Slope pred. intercept: G01, G02, ...	B01, B02, ... B0q
Mean slope:	
G10	B10
Slope pred. slope: G11, G12, G13 ...	B11, B12, .. B1q
Predictor: Wj	Xi
Residual (intercept): U0J	R0I
Residual (slope): U1J	R1I

Some Notes on Hypothesis Testing

Say we look at a random coefficients regression model, with no L-2 predictors, and just linear time as our L-1 predictor, i.e., a fully unconditional model, at L-2. Here is what the model looks like:

L-1: $Y_{ti} = \pi_{0i} + \pi_{1i}a_{ti} + e_{ti}$

L-2: π_{0i} = B00 + R0I

π_{1i} = B10 + R1I

Here is what the hypothesis tests are telling us:

B00 — does the average initial score on Y across persons (communities) differ significantly from zero?

B10 — what was the linear gain in Y units, per time period, per person (community), averaged across persons (communities), and was this significantly different from 0?

R0I — across persons (communities), is the variation in initial scores on Y significantly different from zero?

R1I — across persons (communities), is the variation in linear growth rates on Y significantly different from zero?

What are the reliabilities telling us:

Reliability of: π_{0i} = reliability of estimated true scores of initial status

π_{1i} = reliability of growth rate estimates; measure of "signal to noise" ratio; ratio of variance of empirical Bayes best estimate to the variance of the OLS estimates; reliability of growth parameters for population of persons (communities)